

# Statistics

## Lecture 12



Feb 19-8:47 AM

Consider the chart below

$x$	$P(x)$
1	.1
2	.15
3	.3
4	.25
5	.2

$$1) P(x=3)$$

$$= 1 - [.1 + .15 + .25 + .2] = \boxed{.3}$$

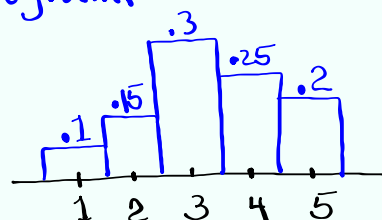
$$2) P(x \geq 2) = 1 - P(x=1)$$

$$= 1 - .1 = \boxed{.9}$$

$$3) P(x \leq 4) = 1 - P(x=5)$$

$$= 1 - .2 = \boxed{.8}$$

4) Draw Prob. dist. histogram.



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Find  $\mu$ ,  $\sigma$ , and  $\sigma^2$ .

$x \rightarrow L1$        $\mu = \bar{x} = 3.3$

$P(x) \rightarrow L2$        $\sigma = \sigma_x = 1.229$

1-Var Stats

List: L1

Freq List: L2

SG 14 ✓

$\sigma^2 = 1.51$

$n=1 \leftarrow$  Total prob.  $\frac{151}{100}$

Vars 5: Statistics 4:  $\sigma_x$

$x^2$  Enter  $\sigma^2 = 1.51$

Math 1:  $\rightarrow$  Frac Enter

$\mu \approx 3$ ,  $\sigma \approx 1$

68% Range  $\mu \pm \sigma \rightarrow [2 \text{ to } 4]$

95% Range  $\mu \pm 2\sigma \rightarrow [1 \text{ to } 5]$

Usual Range

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I sold 25 tickets for \$10 each.  
 one ticket randomly selected  
 the selected ticket wins a calculator worth \$100.

Find expected value per ticket sold.

net	P(net)
10 - 100	$\frac{1}{25}$
10 - 0	$\frac{24}{25}$

net  $\rightarrow$  L1, P(net)  $\rightarrow$  L2  
 use 1-Var Stats with  
 L1 & L2

$\sigma^2 = 384$

E.V. =  $\mu = \bar{x}$

$\$6$

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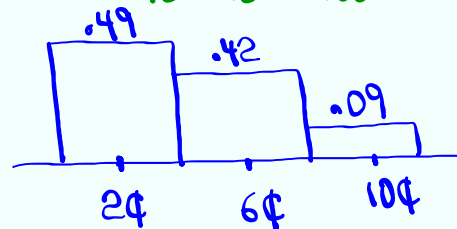
A piggy bank has 7 pennies and 3 nickels.

Randomly take 2 coins with replacements.

Sample space

{	PP	→	2¢	$P(2¢) = \frac{7}{10} \cdot \frac{7}{10} = .49$
	PN	→	6¢	$P(6¢) = 2 \cdot \frac{7}{10} \cdot \frac{3}{10} = \frac{42}{100} = .42$
	NP	→	6¢	
	NN	→	10¢	$P(10¢) = \frac{3}{10} \cdot \frac{3}{10} = \frac{9}{100} = .09$

net	P(net)
2¢	.49
6¢	.42
10¢	.09



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net → L1

P(net) → L2

Use 1-Var Stats

with L1 & L2

$$\mu = 4.4$$

$$n = 1$$

$$\sigma = 2.592$$

$$\sigma^2 = 6.72 = \frac{168}{25}$$

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A piggy bank has 7 pennies and 3 nickels.  
Randomly take 2 coins without replacement.

$$\begin{array}{l}
 PP \quad P(2\phi) = \frac{7}{10} \cdot \frac{6}{9} = \frac{42}{90} \\
 PN \\
 NP \quad P(6\phi) = 2 \cdot \frac{7}{10} \cdot \frac{3}{9} = \frac{42}{90} \\
 NN \quad P(10\phi) = \frac{3}{10} \cdot \frac{2}{9} = \frac{6}{90}
 \end{array}
 \left. \vphantom{\begin{array}{l} PP \\ PN \\ NP \\ NN \end{array}} \right\} \frac{90}{90} = 1$$

L1	L2
2	42/90
6	42/90
10	6/90

$$\mu = 4.4$$

$$\sigma = 2.444$$

$$\sigma^2 = 5.973 = \frac{448}{75}$$

SG 15 ✓

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! Factorial

$$0! = 1$$

$$1! = 1$$

$$n! = n(n-1)(n-2) \cdot (n-3) \cdot \dots \cdot 3 \cdot 2 \cdot 1$$

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

using TI

$$5 \text{ [Math]} \rightarrow \text{PRB} \downarrow \text{[4:]} \text{ [Enter]}$$

Find 12!

$$12 \text{ [Math]} \rightarrow \text{PRB} \downarrow \text{[4:]} \text{ [Enter]}$$

$$479,001,600$$

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$$n C_r = \frac{n!}{r! \cdot (n-r)!}$$

$$5 C_2 = \frac{5!}{2! \cdot (5-2)!} = \frac{5 \cdot 4 \cdot \overset{2}{\cancel{3 \cdot 2 \cdot 1}}}{\cancel{2 \cdot 1} \cdot \cancel{3 \cdot 2 \cdot 1}} = \frac{10}{1} = 10$$

using TI

5 [Math] → PRB ↓ [3:nCr] 2 [Enter]

Find  $7 C_4$

7 [Math] → PRB ↓ [3:nCr] 4 [Enter] [35]

7 different items, there are 35 different combinations to choose 4 of them without replacement and in any order.

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### CA Lotto

choose 5 numbers in any order without replacement from 1 to 50.

1 2 3 4 5  
 1 2 3 4 6  
 1 2 3 4 7  
 ⋮  
 46 47 48 49 50

How many ways can this be done?

$$50 C_5 = 2,118,760$$

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(SG 16)

Binomial Prob. dist.

- 1)  $n$  independent trials.
- 2) Each trial has only two outcomes.
 
$$P(\text{Success}) = p, \quad P(\text{Failure}) = q$$

$$p + q = 1$$

$$q = 1 - p$$

$p$  &  $q$  remain unchanged for all  $n$  trials.
- 3)  $x \rightarrow$  # of successes  
 $n - x \rightarrow$  # of failures

$$P(x) = {}^n C_x \cdot p^x \cdot q^{n-x}$$

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Consider a binomial prob. dist with 5 trials and prob. of success of .6 Per trial.

- 1)  $n = 5$
- 2)  $p = .6$
- 3)  $q = 1 - p = .4$
- 4)  $np = 5(.6) = 3$
- 5)  $npq = 5(.6)(.4) = 1.2$
- 6)  $\sqrt{npq} = \sqrt{1.2} \approx 1.095$
- 7)  $P(4 \text{ Successes})$ 

$$P(x=4) = {}^5 C_4 \cdot (.6)^4 \cdot (.4) = .259$$

$n C_x \cdot p^x \cdot q^{n-x}$   
 $\swarrow \quad \searrow \quad \swarrow \quad \searrow$   
 $n \quad x \quad p \quad q$

$= 5 \times .6 \wedge 4 \times .4 \wedge 1 \text{ [Enter]}$   
 $\rightarrow$

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Consider a binomial Prob. dist. with 10 trials and .4 prob. of success Per trial.

1)  $n=10$       2)  $p=.4$       3)  $q=1-p=.6$

4)  $np=10(.4)=4$       5)  $npq=10(.4)(.6)=2.4$       6)  $\sqrt{npq}=\sqrt{2.4}=1.549$

7)  $P(5 \text{ Successes})$   
 $P(X=5) = {}^{10}C_5 \cdot (.4)^5 \cdot (.6)^5 \approx .201$

${}^n C_x \cdot p^x \cdot q^{n-x}$   
 using TI

2nd VARS ↓ binompdf

Trials 10  
 P: .4  
 X Value: 5  
 (Paste) (Enter)

Your work  
 $P(X=5) = \text{binompdf}(10, .4, 5) = .201$

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Suppose we flip a fair coin 100 times and success is to land tails.

1)  $n=100$       2)  $p=.5$       3)  $q=.5$

4)  $np=100(.5)=50$       5)  $npq=100(.5)(.5)=25$       6)  $\sqrt{npq}=\sqrt{25}=5$

7)  $P(\text{exactly } 50 \text{ tails})$   
 $P(X=50) = \text{binompdf}(100, .5, 50) = .080$

8)  $P(\text{at most } 50 \text{ tails})$   
 $P(X \leq 50) = P(X=50) + P(X=49) + \dots + P(X=0)$   
 $= \text{binomcdf}(100, .5, 50) = .540$

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You are guessing on every question of a multiple-choice exam with 100 questions. Each question has 5 choices but only one correct choice.

1)  $n = 100$       2)  $p = \frac{1}{5} = .2$       3)  $q = \frac{4}{5} = .8$

4)  $np = 100(.2) = \boxed{20}$       5)  $npq = 100(.2)(.8) = \boxed{16}$       6)  $\sqrt{npq} = \sqrt{16} = \boxed{4}$

7)  $P(\text{guess exactly 25 Correct Answers})$   
 $P(x=25) = \text{binom.pdf}(100, .2, 25) \approx \boxed{.044}$

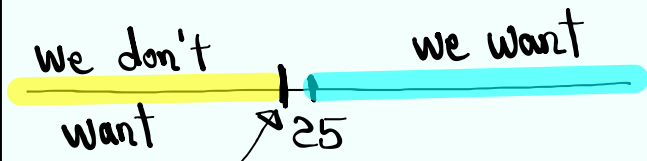
8)  $P(\text{guess at most 25 Correct Answers})$   
 $P(x \leq 25) = \text{binomcdf}(100, .2, 25) = \boxed{.913}$

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9)  $P(\text{guess at least 25 Correct answers})$

$P(x \geq 25) = 1 - P(x \leq 24)$

Total Prob.  $\swarrow$



$= 1 - \text{binomcdf}(100, .2, 24)$

$= \boxed{.131}$

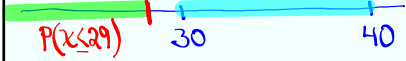
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64 newborn babies were randomly selected  
 Suppose success is having a girl.

1)  $n = 64$       2)  $p = .5$       3)  $q = .5$

4)  $np = 32$       5)  $npq = 16$       6)  $\sqrt{npq} = \sqrt{16} = 4$

7)  $P(\text{having between } 30 \text{ \& } 40 \text{ girls, inclusive})$

$$P(30 \leq x \leq 40) = P(x \leq 40) - P(x \leq 29)$$


$= \text{binomcdf}(64, .5, 40) - \text{binomcdf}(64, .5, 29)$

$= .717$

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